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Neighboring chromaticity influences how white a surface looks

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ABSTRACT

To identify surface properties independently of the illumination the visual system must make assumptions about the statistics of scenes and their illumination. Are assumptions about the intensity of the illumination independent of assumptions about its chromaticity? To find out, we asked participants to judge whether test patches within three different sets of surrounding surfaces were white or grey. Two sets were matched in terms of their maximal luminance, their mean luminance and chromaticity, and the variability in their luminance and chromaticity, but differed in how luminance and chromaticity were associated: the highest luminance was either associated with colorful surfaces or with achromatic ones. We found that test patches had to have a higher luminance to appear white when the highest luminance in the surrounding was associated with colorful surfaces. This makes sense if one considers that being colorful implies that a surface only reflects part of the light that falls on it, meaning that the illumination must have a higher luminance (a perfectly white surface reflects all of the light falling on it). In the third set, the colorful surfaces had the same luminance as in the set in which they were associated with the highest luminance, but the achromatic surfaces had a lower luminance so that the overall mean luminance was lower. Despite the constraints on the illumination being identical, test patches did not have to have as high luminance to appear white for the third set. Considering the layout of the surfaces in the surrounding revealed that test patches did have to have the same high luminance if the high luminance colorful surfaces were adjacent to the target patch. Thus, the assumptions about the possible illumination are applied locally. A possible mechanism is relying on the contrast within each type of cone: for a surface to appear white it must stimulate each of the three kinds of cones substantially more than do any neighboring surfaces.

1. Introduction

Our judgments about the material of which objects are made relies on an interaction between the reflectance of the objects' surfaces and the intensity, color and geometry of the illumination (Fleming, 2014). Natural scenes contain diverse objects and illuminations, making it impossible to judge surface properties (such as color) without making assumptions about the regularity of the world (Shevell & Kingdom, 2008). In some cases the assumptions made by the visual system can vary across individuals or across time, giving rise to spectacular illusions such as that generated by the #TheDress (Brainard & Hurlbert, 2015; Gegenfurtner, Bloj, & Toscani, 2015), which has received much interest both from the media and from the scientific community. Nevertheless, the colors in most complex scenes are perceived quite consistently across individuals and more or less independently of the color of the illumination. This is known as color constancy (Foster, 2011).

The visual system must judge the chromaticity and saturation of surfaces from the ratio of stimulation of different types of cones (Brenner, Granzier, & Smeets, 2007; Foster & Nascimento, 1994; Land & McCann, 1971). It might also evaluate complex chromatic properties of the image to mitigate the influence of variations in the illumination on perceived surface color (Brainard et al., 2006; Golz & MacLeod, 2002). For any given illumination there is a physical limit to the combination of luminances and chromaticities that can arise by diffuse reflection alone because reflection can only reduce the intensity of the light at each wavelength. The set of all possible combinations produces the theoretical object-color set of the illuminant, as developed early in the 20th century (Kuehni & Brill, 2010; Schrödinger, 1920). The corresponding chromaticities were computed by David L. MacAdam to obtain the MacAdam limits (MacAdam, 1935). As a result of these physical properties, the correlation between the color and luminance of light reflected from surfaces contains information about the illumination. Previous studies have considered that the visual system may make

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use of such correlations to achieve color constancy (Golz & MacLeod, 2002; Granzier, Brenner, Cornelissen, & Smeets, 2005). Light that does not arise from diffuse reflection of the illumination (highlights; fluorescence; Rayleigh or Mie scattering; Nassau, 1987) or light from additional light sources does not necessarily comply to these theoretical limits, but such light is rarely encountered in nature (Linhares, Pinto, & Nascimento, 2008).

Besides having to consider the color of the illumination, it is also necessary to consider its intensity to fully estimate surface reflectance. This is necessary to distinguish a grey object illuminated by bright light from a white object illuminated by less bright light. For simple achromatic scenes the surface that corresponds with the highest intensity of light reaching the eye is assumed to be white (Gilchrist & Radonjić, 2009; Li & Gilchrist, 1999) but in complex scenes it is not always that simple (Gilchrist et al., 1999; Kingdom, 2011). One factor that might reveal that the illumination is brighter than one would infer from the highest intensity of light in a scene is that the chromaticity of the surface with the highest intensity imposes an additional constraint. If the highest luminance in the light reflected from a scene is from a surface that is clearly white, the luminance of the light reflected from that surface provides a reasonable estimate of the intensity of the illumination. However, if the highest luminance from the scene is from a surface that, for instance, is clearly blue, the intensity of the illumination must be higher than the luminance of the light reflected from that surface, because the surface does not reflect all the light falling on it at longer wavelengths (which is why it is blue). In other words, the maximal perceived saturation for purely reflecting surfaces can only be obtained for middle lightness, so the luminance of saturated colored surfaces in a complex scene can be used to judge the level of illumination even when such surfaces do not have the highest luminance in the scene, and reliable estimates can be obtained even when there is no truly white surface available. It is known that such considerations of what is physically possible determine when patches in a scene stop looking like illuminated surfaces and become self-luminous (Speigle & Brainard, 1996).

The goal of this work was to test whether the relationship between luminance and chromaticity influences our judgments of surface lightness. This was done by determining the luminance at which subjects report a transition between grey and white for a target patch in scenes with identical distributions of luminance and chromaticity, but different relationships between luminance and chromaticity. We also investigated whether such an effect is global or local in nature. We show that subjects do consider the relationship between luminance and chromaticity: a higher luminance is required to perceive surfaces as white when the highest intensity of light in the surrounding comes from colored surfaces. The data suggests that an important part of this effect is determined by the directly adjacent surfaces.

2. Methods

2.1. The stimuli

Stimuli were computer simulations of patterns of flat colored mate surfaces illuminated by uniformly diffuse light. The patterns were made of 12×8 colored squares (Fig. 1). Each square subtended 1° of visual angle at the viewing distance of 1.8 m. The patterns were displayed on a 21-inch Apple Studio Display calibrated in color and luminance with a telespectroradiometer (SpectraColorimeter, PR-650, PhotoResearch Inc., Chatsworth, CA). There were three types of patterns: Standard, Colorful and Darker. All three patterns consisted of 48 grey and 48 colored (16 red, 16 green and 16 blue) squares. The highest luminance was always 36 cd/m^2. In the Standard pattern the eight squares with the highest luminance were grey (Fig. 1A). In the Colorful and Darker patterns they were green (Fig. 1B and C).

In the Colorful pattern there were light and dark green squares (36 and 12 cd/m^2; 8 each), light and dark red squares (17 and 4.2 cd/m^2; 8 each), light and dark blue squares (5.2 and 1.6 cd/m^2; 8 each), and three kinds of grey squares (24, 8.4 and 3.3 cd/m^2; 16 each). The greys had a 1931 CIE$_{xy}$ chromaticity of (0.28, 0.29). The colors were generated by stimulating only one of the three primaries: red (0.62, 0.53), green (0.26, 0.6) or blue (0.15, 0.06).

The Darker pattern was obtained from the Colorful pattern by reducing the luminance of the grey surfaces by half. Thus, this pattern has the same peak luminance for squares of each color, but a lower average luminance (bar charts in Fig. 1B and C).

The Standard pattern was obtained from the Colorful pattern by replacing each colorful square by a grey square with the same luminance, and each grey square by a colorful square with the same luminance, making sure that each color is represented equally often. Thus, the patterns are matched in average luminance and chromaticity. They are also matched in the variability in luminance and chromaticity. However, they differ in the kind of square (grey or colored) that has the highest luminance (bar charts in Fig. 1A and B).

2.2. Procedure

Each observer took part in four sessions. Each session had 30 blocks of 15 trials, with 10 blocks for each pattern. The blocks of each pattern were presented in random order. An achromatic target circle with a diameter of 1° of visual angle was superimposed on the pattern, 2 s after the pattern appeared. The circle was always centered at the intersection of four squares (i.e. at a corner of all four squares) but never at an intersection that included the outmost row of squares (see Fig. 1). Its position was selected at random for each trial, as was the pattern of squares. Observers had to indicate whether the target circle was grey or white by pressing the 'g' or 'w' key of a computer keyboard. A separate staircase procedure was used to find the luminance at which observers' judgments switched from grey to white for each of three kinds of patterns. The luminance increased by one step if the observer responded 'grey' and decreased by one step if the observer responded 'white'. The step was quite small, but we added some additional variability to the tested luminance values by shifting the luminance by 5 steps in a random direction once every 10 trials to keep observers motivated. The room was dark except for the light from the screen.

2.3. Analysis

The critical luminance was determined for each observer and pattern by fitting a cumulative normal distribution to the observer's judgments (grey or white) as a function of log target luminance using the method proposed by Wichmann and Hill (2001). The luminance value at which the fit suggested that subjects were equally likely to answer grey or white was considered to be their threshold for that pattern. This threshold is quite subjective, because participants are free to pick a luminance (or implied reflectance) from which they are willing to call a surface white rather than grey, but assuming that this criterion does not differ for the different patterns we consider it valid to use the difference between the thresholds for the different patterns as a measure of how the pattern influences the perceived surface lightness. We therefore determined how the thresholds for the Colorful and Darker patterns differed from the threshold for the Standard pattern. We determined these differences for each observer and report the mean difference with the standard error across observers. We also followed this whole procedure after selecting trials in which various numbers of the squares that were directly adjacent to the target were either both bright and colorful (for the Colorful and Darker patterns) or were the grey squares with the equivalent luminance (for the Standard pattern).

2.4. Participants

Ten observers (including one of the authors) performed the experiment. All except the author were naïve to the purpose of the

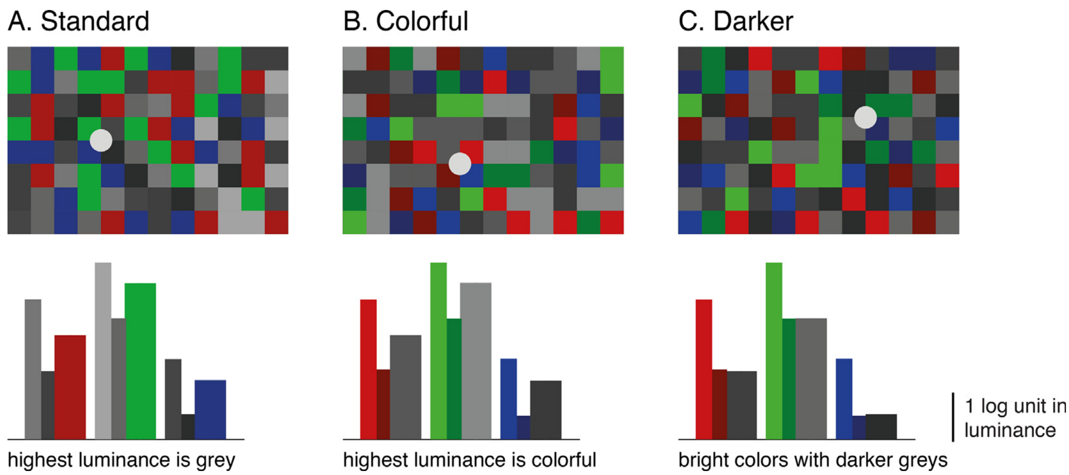

Fig. 1. Example of the appearance and luminance histograms of the three kinds of test patterns: Standard (A), Colorful (B) and Darker (C). The Standard and the Colorful patterns are matched in terms of luminance and color distribution, and even in terms of the average luminance of the colored squares. The critical difference is that some colored squares in the Colorful pattern have a particularly high luminance. If the visual system considers that the illumination must be brighter for a surface to reach a given luminance if the surface is colorful, surfaces should look darker when presented on the Colorful pattern (the illumination is judged to be brighter). The Darker and the Colorful patterns have the same bright colorful squares, so the presumed illumination is constrained in the same manner, but the luminance of the grey surfaces is lower for the Darker pattern. This luminance should be irrelevant if the overall constraints on the illumination are considered. A target circle was superimposed on the patterns to estimate the luminance at which observers' judgments switched from grey to white. This was done for each of the three kinds of patterns using three separate staircases. The upper part of each panel gives an impression of what the stimulus looked like. The lower part indicates the log luminance for each of the 12 kinds of squares. The bars are split into three groups, one for each color, to make it easier to see how various measures were equated. Within each group two of the bars are identical, so actually there are only 9 different kinds of squares but three of them appear twice as often as the rest.

experiment. Their color vision was normal as tested with a Rayleigh anomaloscope (Oculus Heidelberg Multi Color), Ishihara plates and Farnsworth-Munsell 100 hue test. The experiments were performed in accordance with the tenets of the Declaration of Helsinki and informed consent was obtained from all observers.

3. Results

Fig. 2 shows the data of one of the observers for all three patterns. The curves are fits of cumulative normal distributions to the fraction of white responses. The threshold is the luminance at which the curves cross 0.5 (horizontal white line). The curve for the Colorful pattern (and also, but less so, for the Darker pattern) is clearly shifted to the right compared to that for the Standard pattern, showing that the observer required a higher intensity to reach the same judgment for the target on the Colorful pattern than for the one on the Standard pattern. The horizontal separation at the level of the white line is our measure of the influence of the pattern.

The red bar in Fig. 3 shows the average of the differences between the observers' luminance thresholds for the Colorful and Standard patterns. Considering the standard error of these differences we can conclude that the test disk's appearance was not the same for the two patterns. The black bar shows the average difference between thresholds for the Darker and Standard patterns. In this case the error bars

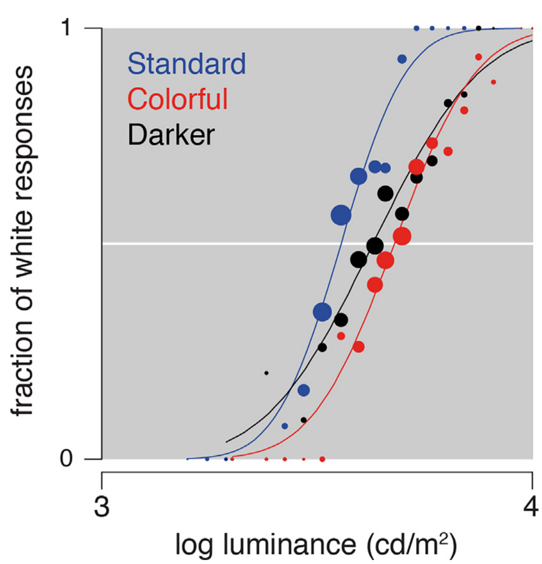

Fig. 2. Data of one of the observers for the three patterns. The points show the fraction of times that the observer pressed the 'w' key (to indicate that the target looked white) as a function of the natural logarithm of the target circle's luminance. The point size indicates the number of times that a luminance value was presented. Curves show fits of cumulative normal distributions to the fraction of white responses. The threshold for each pattern is the luminance at which the curve reaches a fraction of 0.5 (white line).

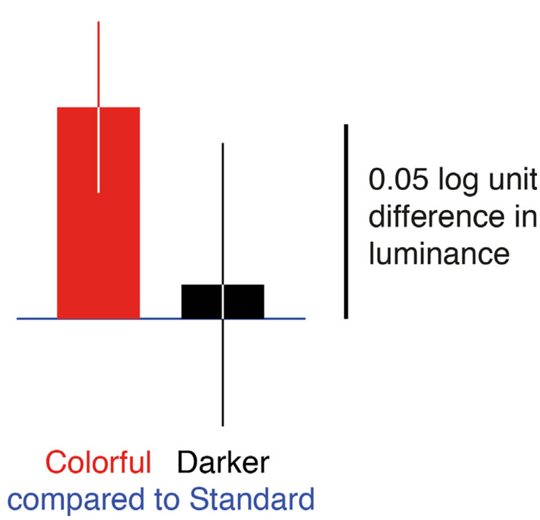

Fig. 3. Average differences across observers between the luminance thresholds for Colorful and Standard patterns (red bar) and for Darker and Standard patterns (black bar). Error bars are standard errors across observers. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

show that the test disk's appearance did not differ systematically when presented on the two patterns. A higher luminance was required for the test disk to appear white on the Colorful pattern than for it to appear to be white on the Standard pattern, despite the luminance and chromaticity being matched in so many ways between the two patterns. This is consistent with the idea that the correlation between luminance and chromaticity influences the estimated illumination in accordance with what is physically possible. However, for the Darker pattern the constraints on the illumination are the same as for the Colorful pattern, but the threshold was not influenced in the same manner. The Darker pattern only differs from the Colorful pattern in that the greys have a lower luminance. Thus, reducing the luminance without changing the constraints on the illumination reduces the luminance required for the test disk to appear white.

When considering the equivalence between the Darker and Colorful patterns we assumed that all the squares of the pattern contribute equally to the judged lightness of the target. But is that really so? The squares adjacent to the target are more likely to be illuminated by the same light source than ones that are far away, so maybe they should be given more weight. To examine this possibility we segmented the data by the number of surfaces adjacent to the test disk that had bright colors: none, one, two or more. The Standard pattern was segmented by the number of grey surfaces adjacent to the test disk that had the same luminance as these bright colors. The results are shown in Fig. 4. When none of the four squares that were partially covered by the target were the brightest representative of one of the three colors, the target still appeared to require a little more luminance to look white for the Colorful pattern than for the Standard pattern, so the required luminance may not only depend on the directly adjacent surfaces. However, the difference in required luminance was larger when there were more bright colored surfaces close to the test disk, suggesting that there is a strong local effect.

When there were two or more bright colored surfaces near the test disk, the required luminance for the target to appear to be white was similar for the Dark and Colorful patterns. This is what would be expected if the possible illumination was estimated from the combination of chromaticity and luminance close to the test region, because a lower luminance can always be attributed to less reflection from a surface but a higher luminance requires a more intense illumination. In accordance with the required luminance mainly depending on the nearby squares' luminance and chromaticity for the Colorful pattern, the lower average luminance of the squares in the Darker pattern decreased the required luminance when none of the adjacent squares were bright and colorful.

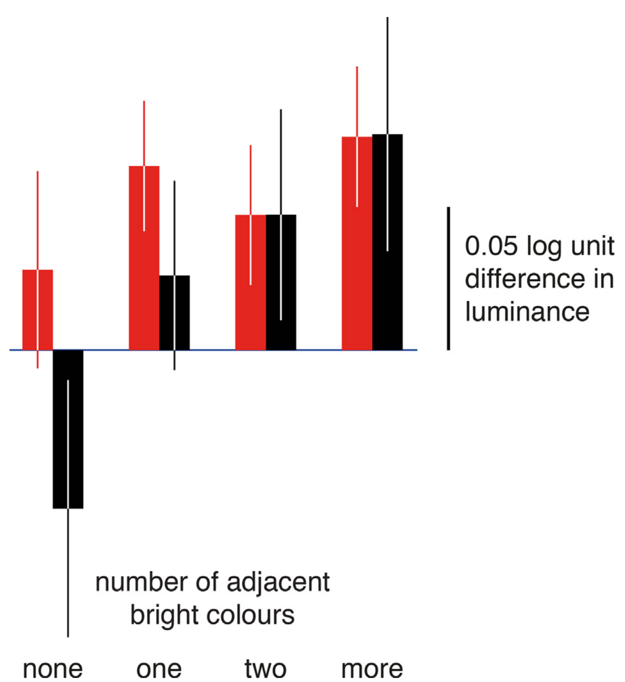

Fig. 4. The same analysis as that shown in Fig. 3, after segmenting the data by the number of surfaces directly adjacent to the test disk that had bright colors. The results are shown for none, one, two or more adjacent bright colors. Red bars indicate the difference between Colorful and Standard patterns and black bars the difference between Darker and Standard patterns. Error bars are standard errors across observers. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Conclusions and discussion

A comparison between the luminance needed for the target to look white on the Colorful and Standard patterns supports the idea that the fact that reflecting surfaces look colorful because they do not reflect all the light that illuminates them is considered when making assumptions about the intensity of the illumination to interpret local luminance in terms of surface reflectance. A higher luminance was needed for the target to look white on the Colorful pattern than on the Standard pattern, despite the two patterns having the same luminance distributions and the same average stimulation for each cone type. Although the colors (ratios of cone stimulation) and the luminances were both matched, the maximal stimulation of each cone type was not matched. This is most evident when considering that the bright green square had the highest luminance in the Colorful pattern. The M − cone stimulation was therefore highest for this square. It was obviously higher than for any other square within the Colorful pattern, but it was also higher than for any square in the Standard pattern. In the Standard pattern, the brightest green square had a lower luminance and the square with the same, highest luminance was grey rather than green.

Although the comparison between the Colorful and Standard patterns supported our proposal that the chromaticity is considered when evaluating the intensity of the illumination, a comparison with the Darker pattern does not. Only reducing the luminance of the grey squares in the Darker pattern (with respect to the Colorful pattern) gave this pattern a lower average luminance without changing the highest luminance or its relation with chromaticity, and therefore without changing the minimal level of illumination that must have been present or the highest stimulation of each kind of cone. Nevertheless, less luminance was required for the target to look white in this case. The required luminance was somewhere between that for the Colorful and Standard patterns, confirming that the color of the bright surfaces influences the judgments of the intensity of the illumination, but the luminance of the darker surfaces did matter.

Splitting the data into trials in which there happened to be bright colorful squares immediately adjacent to the target and ones in which there happened not to be such squares at those four positions revealed that the layout made a difference. This difference was particularly evident for the Darker pattern. When the same bright colorful squares were next to the target, the target switched from looking grey to looking white at the same luminance as for the Colorful pattern. When the bright colorful squares were further away the target looked lighter (less luminance was required for it to look white), in accordance with the surrounding surfaces being darker (on average). Thus, although we find confirmation for our idea that since reflecting surfaces look colorful because they do not reflect all of the light that falls on them the illumination is assumed to be higher when the highest luminance is colorful, this effect appears to be localized in space. Spatial localization of assumptions about the illumination makes sense in complex scenes in which the illumination cannot be considered to be uniform, such as in many natural scenes (Gilchrist & Radonjic, 2010; Nascimento, Amano, & Foster, 2016). In our simple configuration it is perhaps somewhat surprising, but the idea that the illumination is only assumed to be uniform within small regions, even in quite simple scenes, is not new (Gilchrist, 2018; Land & McCann, 1971; Radonjić & Gilchrist, 2013).

The effects reported here might be related to the fact that color appearance depends on the chromatic variance of the surrounds (Brown & MacLeod, 1997), an effect that works as a gamut compression for surrounds with large chromatic variance, which is also largely local in origin (Granzier et al., 2005). It is consistent with models that estimate the illumination on the basis of the balance of the colors present in the scene and individual assumptions about the distribution of natural colors (Morimoto, Fukuda, & Uchikawa, 2016; Uchikawa, Fukuda, Kitazawa, & MacLeod, 2012). The mechanism for scaling cone stimulation when judging lightness is somewhat analogous to the use of the MaxRGB method for illuminant estimation (Funt & Shi, 2010). Such a mechanism would also explain why white surfaces look grey when the illumination is designed to artificially enhance the perceived chromaticity of a scene (Nascimento & Masuda, 2012; Thornton, 1974): under such narrowband illumination white surfaces are not necessarily as much lighter than surfaces with very saturated colors as would normally be the case.

Whatever the mechanism, our study confirms that some physical regularity of natural scenes derived by the way light is reflected by natural pigments is considered in the way we interpret the light reaching our eyes, as indeed it should be if we want to be able to rely on vision to recognize objects by their surface properties despite varying illumination. In the above we only consider diffusely reflecting surfaces. Specular reflectance can make a small area on the surface of a white shiny mug provide the eye with a much higher luminance than is provided by the rest of the surface of the mug, and yet the whole mug looks white. Thus, the situation is more complicated when the illumination is evidently not uniform (Toscani, Valsecchi, & Gegenfurtner, 2013, 2017). Nevertheless, the importance of the relationship between luminance and color implies that it may be worthwhile to study lightness judgments in colorful scenes, rather than isolating lightness from color by studying the former in black and white images.

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